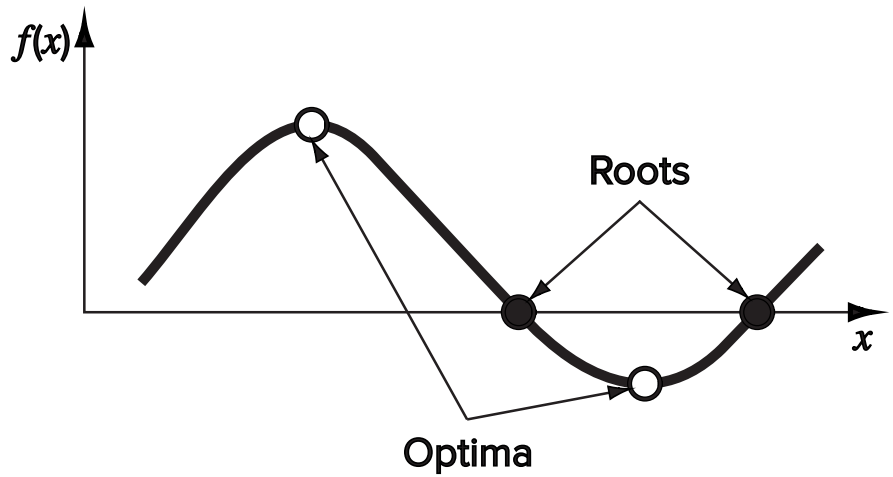
ENGR 504: Course Introduction (Spring 2026)

- mathematical problems
- scientific computing
- course information

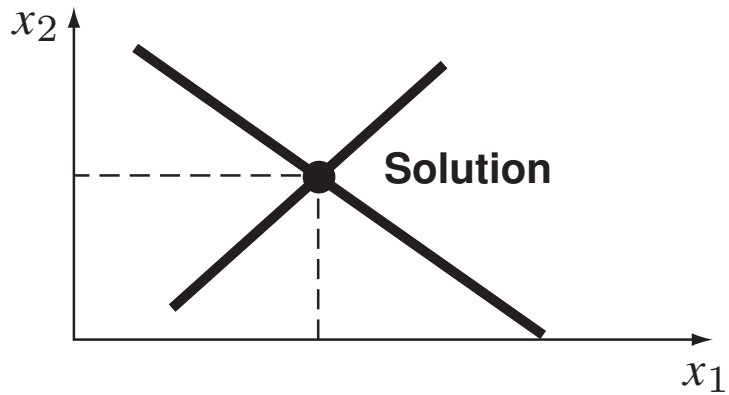
Roots and optimization

- roots: solve $f(x) = 0$ for x
- optimization: find x that minimize or maximize $f(x)$



Linear equations

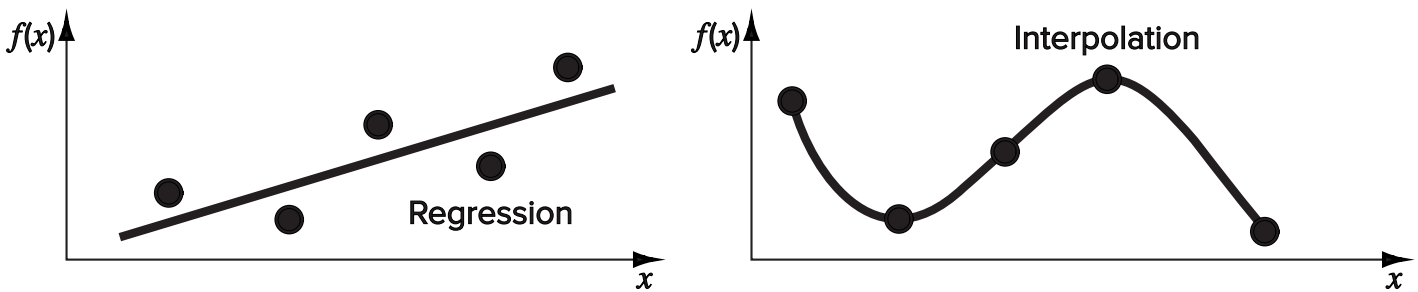
solve $Ax = b$ where A is a matrix and b is a vector



$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

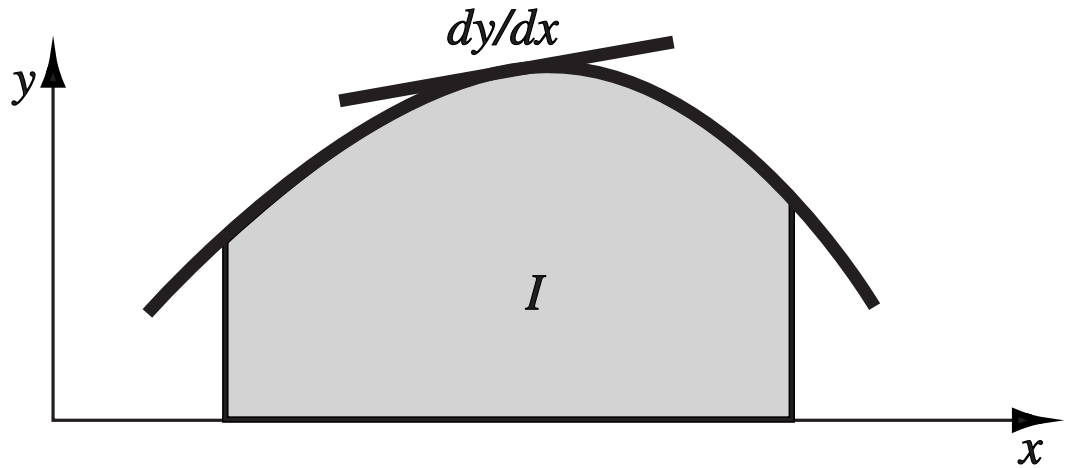
Curve fitting: regression and interpolation

- regression: find $f(x)$ that 'best' fit a given points
- interpolation: find $f(x)$ that exactly passes through given points



Integration and differentiation

- integration: find area under the curve $I = \int_a^b f(x)dx$
- differentiation: find slope of curve $\frac{dy}{dx}$



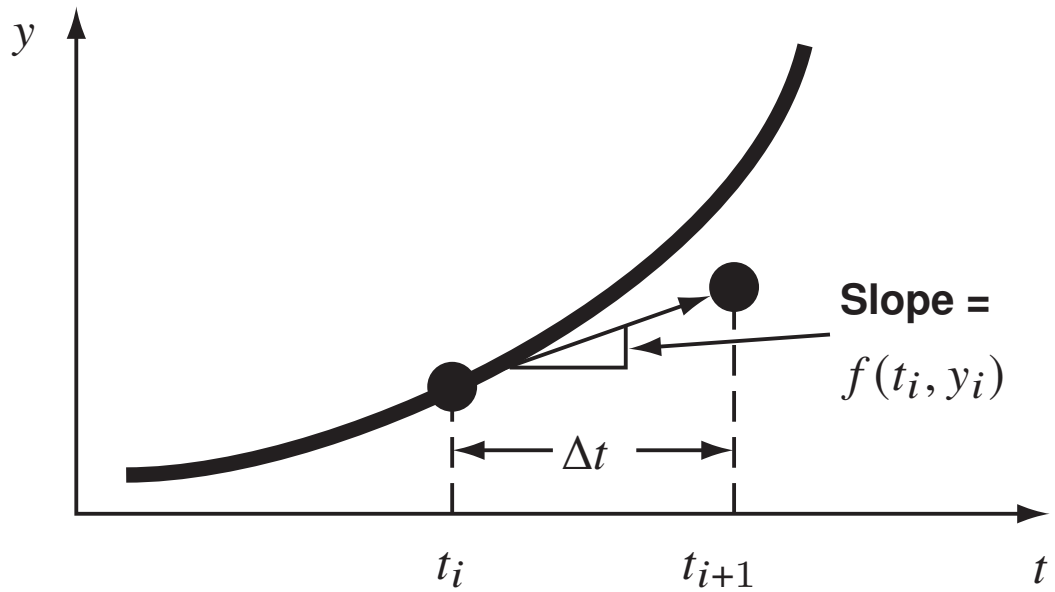
Ordinary differential equations

given

$$\frac{dy}{dt} = f(t, y)$$

solve for y as a function of t

$$y_{i+1} = y_i + f(t_i, y_i)\Delta t$$

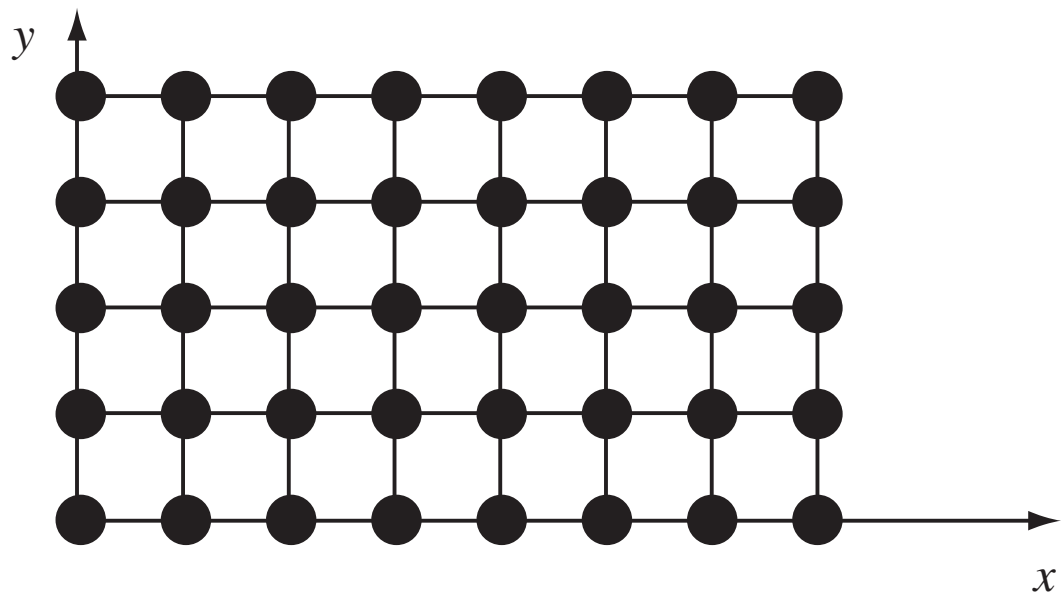


Partial differential equations

given

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

solve for u as function of x, y



Outline

- mathematical problems
- **scientific computing**
- course information

Scientific computing

Scientific computing

- scientific computing involves developing and studying *numerical algorithms* for solving mathematical problems in various scientific and engineering fields
- most mathematical problems cannot be solved exactly (some can)
- use an iterative algorithm that ultimately converges to a solution

Numerical errors: numerical computing involves the presence of errors

- results of computations are approximate
- goal: ensure the resulting error is tolerably small

Problem solving process

- mathematical models formulated to explain observed phenomena
- develop algorithms for efficient, accurate, and reliable solutions
- implement algorithm in a computer to simulate the physical process numerically
- interpret and validate the computed results

General strategy for solving problems

replace a difficult problem with an easier one that has same or closely related solution

- replacing in infinite-dimensional spaces with finite-dimensional spaces
- replacing infinite processes with finite processes, such as replacing integrals or infinite series with finite sums, or derivatives with finite differences
- replacing differential equations with algebraic equations
- replacing nonlinear problems with linear problems
- replacing complicated functions with simple functions, such as polynomials
- replacing general matrices with matrices having a simpler form

Problem solving environment

high-level languages for numerical computing:

- MATLAB
- Julia
- Python
- R
- ...

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Course topics

- numerical errors and floating point numbers
- vectors and matrices with their practical examples
- numerical methods for solving linear equations
- linear and nonlinear least squares
- nonlinear equations and optimization
- eigenvalue problems
- interpolation
- numerical differentiation and integration
- numerical methods for solving differential equations
- applications in engineering, finance, data analysis,...

Course information

Useful references

- S. Boyd and L. Vandenberghe. *Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares*. Cambridge University Press, 2018. https://web.stanford.edu/~boyd/vmls/
- U. M. Ascher. *A First Course on Numerical Methods*. Society for Industrial and Applied Mathematics, 2011.
- M. T. Heath. *Scientific Computing: An Introductory Survey* (revised second edition). Society for Industrial and Applied Mathematics, 2018.
- R. L. Burden and J. D. Faires. *Numerical Analysis* (9th edition). Brooks/Cole, Cengage Learning, 2011.
- S. C. Chapra. *Applied Numerical Methods with MATLAB for Engineers and Scientists* (5th edition). McGraw Hill, 2023.

Grading

- Homework (50%)
- Midterm exam (20%)
- Final exam (30%)

weights are approximate, and we reserve the right to adjust them if necessary

(see syllabus on Moodle for detailed information)